Home Work 3 : Fluid Dynamics AE2202

March, 4th 2020

Solve all problems

1.

Consider a steady, two-dimensional, incompressible flow of a newtonian fluid in which the velocity field is known, i.e., $u = -2xy$, $v = y^2 - x^2$, $w = 0$. (*a*) Does this flow satisfy conservation of mass? (*b*) Find the pressure field, $p(x, y)$ if the pressure at the point $(x = 0, y = 0)$ is equal to p_a.

2.

A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as in Fig. **3** The velocity profile is

$$u = Cy(2h - y) \qquad v = w = 0$$

Find the constant C in terms of the specific weight and viscosity and the angle θ. Find the volume flux Q per unit width in terms of these parameters.

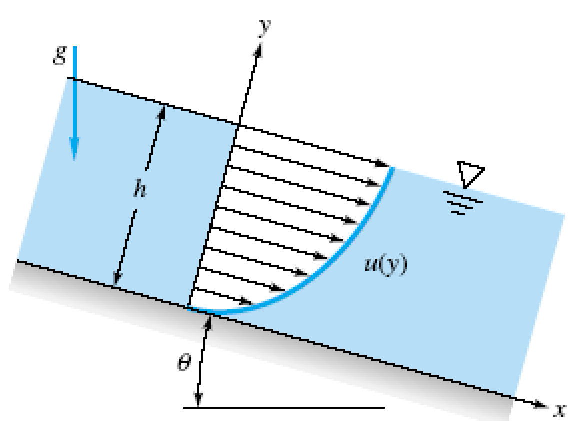


Figure 3

3.

A viscous liquid of constant ρ and μ falls due to gravity between two plates a distance $2h$ apart, as in Fig. **4.** The flow is fully developed, with a single velocity component $w = w(x)$. There are no applied pressure gradients, only gravity. Solve the Navier-Stokes equation for the velocity profile between the plates.

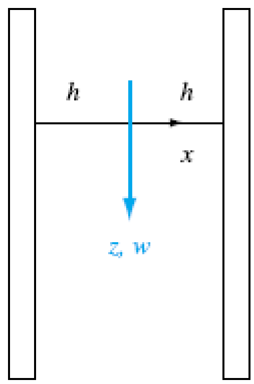
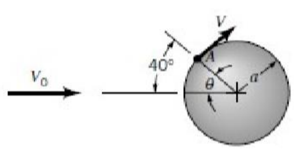


Figure 4

4. A fluid flows past a sphere with an upstream velocity of V_o = 40 m/s as shown in figure below. From a more advanced theory it is found that the speed of fluid along the front part of sphere is V = 3/2 V_o sin θ.



$$\mathbf{a} = V\frac{\partial V}{\partial s}\hat{\mathbf{s}} + \frac{V^2}{\mathcal{R}}\hat{\mathbf{n}} \quad \text{or} \quad a_s = V\frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{\mathcal{R}}$$

Figure 5

a. Determine the **streamwise** and **normal components** of acceleration at point A, if radius of sphere is a = 0,20 m. Use coordinate streamline.

b. Plot a graph of the streamwise acceleration a_s, the normal acceleration a_n, and the magnitude of the acceleration as function of θ for 0 < θ < 90°. What point is the acceleration a maximum and a minimum.

5. A two-dimensional velocity field is given by

$$V = (x^2 - y^2 + e^{2x})i - (2xy + y)j$$

in arbitrary units. At $(x, y) = (2, 3)$, compute (a) the accelerations a_x and a_y, (b) the velocity component in the direction θ = 33°, (c) the direction of maximum velocity, and (d) the direction of maximum acceleration

6. Consider a sphere of radius R immersed in a uniform stream U_o, as shown in **Figure 6**. The fluid velocity along streamline AB is given by

$$V = ui = Uo(1 + \frac{R^3}{x^3})i$$

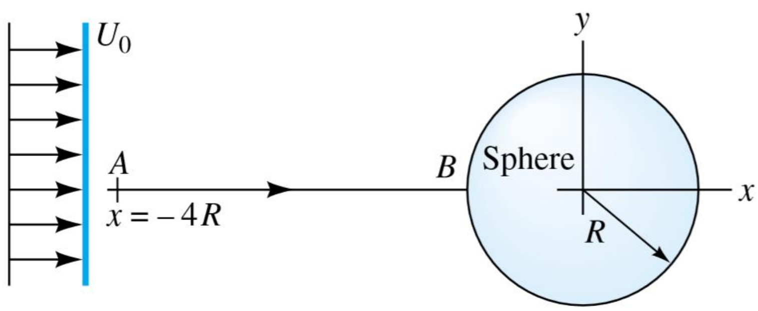


Figure 6

Find:

a. The position of maximum fluid acceleration along AB and

b. The time required for a fluid particle to travel from A to B.

7. Flow through the converging nozzle in **Figure 7** can be approximated by the one-dimensional velocity distribution:

$$u \approx Vo\left(1 + \frac{2x^2}{L^2}\right) \qquad v \approx 0 \qquad w \approx 0$$

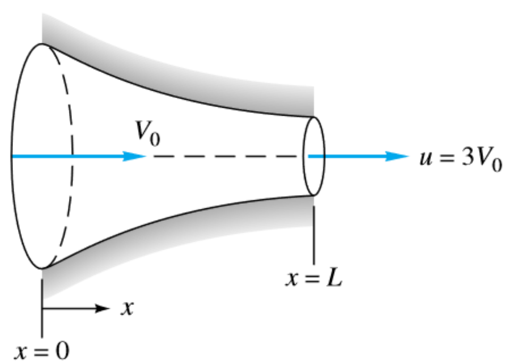


Figure 7

a. Find a general expression for the fluid acceleration in the nozzle

b. For the specific case V_o = 3 m/s and L = 1 m , compute the acceleration, in g's, at the entrance and at the exit

8. The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - \frac{z^2}{2} + 4$$

a. Determine the volumetric dilatation rate and interpret the results

b. Determine an expression for the rotation vector. Is this an irrotational flow field?

9. The two-dimensional velocity field for an incompressible Newtonian fluid is described by the relationship

$$\vec{V} = (12xy^2 - 6x^3)\hat{\imath} + (18x^2y - 4y^3)\hat{\jmath}$$

Where the velocity has units of m/s when x and y are in meters. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} at the point x = 0.7, y = 1.2 m if pressure at this point is 5 kPa and the fluid is glycerin at 20 °C. Show these stresses on a sketch. (Recall: $\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}$).

10. The velocity components for an incompressible, plane flow are

$$v_r = Ar^{-1} + Br^{-2}\cos\theta$$

$$v_\theta = Br^{-2}\sin\theta$$

Where A and B are constants. Determine the corresponding stream function.

Good Luck